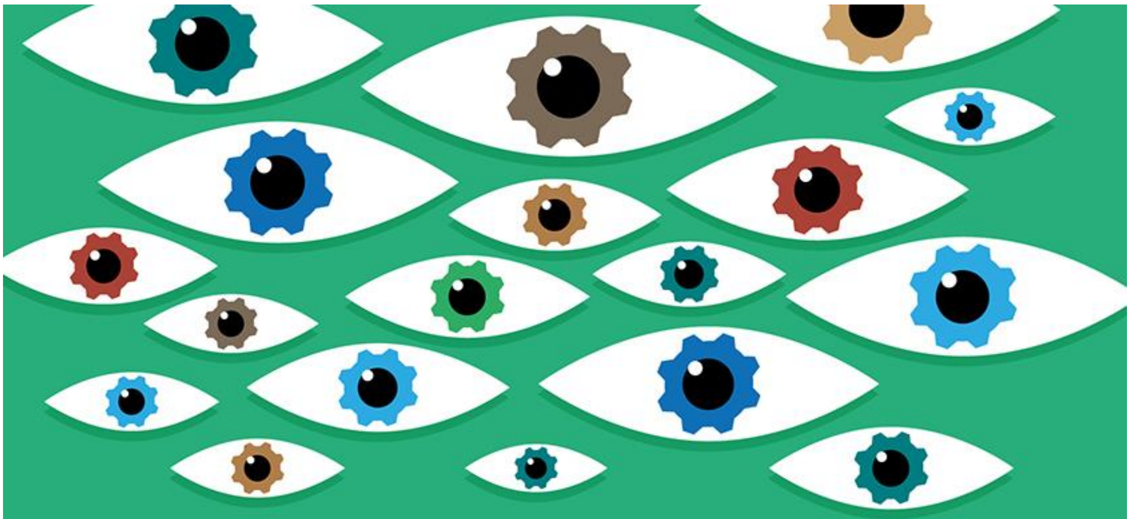

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Appraising Freedom of Choice During Political Elections: A Theoretical Approach

Chantós Mariani

Abstract

Information about candidates' political platforms is of the utmost importance to a well-considered vote and to a proper exercise of political rights. Nevertheless, candidates frequently adopt ambiguous positions on relevant issues, leading voters to see candidates as lotteries over the ideological space, instead of well-defined policy alternatives. Hence, this paper discusses how uncertainty can affect voters' perception of freedom of choice during political elections using the theoretical background of the Freedom of Choice Literature (FCL), originated in Suppes (1987), Sen (1988) and Pattanaik and Xu (1990). Two distinct axiomatic approaches of freedom, the *simple-cardinality-based ordering* of Pattanaik and Xu (1990) and the *known-option-based rule* of Arlegi and Dimitrov (2005) are compared, where some refinements are proposed to a proper assessment of voters' uncertainty level. Some conclusions are then presented, and counterintuitive results that arise from these approaches are discussed, where we highlight that voters' tolerance about uncertainty, combined with the sort of information made available to voters by political campaigns, may lead to a severe curtailment of voters freedom of choice during the choice procedure.

Keywords: Freedom of Choice. Elections. Uncertainty. Ambiguity.

1 Introduction

Freedom to choose and to exercise political rights during an election can be seen as a central element for a life worth of living. Sen (1999), when discussing the reasons people have to value democracy, pinpoints the *constructive importance* that democratic regimes exert in the definition of needs, rights and duties, and how this fosters a proper evaluation of which available candidates may fulfill more effectively those needs. In this context, information about candidates' political platforms is of the utmost importance to the promotion of public discussion, the reaching of a well considered decision, and to the effective expression of one's own political opinions.

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The importance of this element, however, had been neglected by traditional microeconomics theory until the recent emergence of the so-called *freedom of choice* literature. As synthesized in Gaertner and Xu (2011), representing individuals' choices by sets of indifference curves (that is, in the usual microeconomic sense), may not be adequate to reflect accurately the richness of opportunities that these agents are experiencing while consummating the choice. In other words, besides the chosen set of goods, the *choice set*, which is composed by all available options in the choice process, and the freedom that it provides to the individual, is also a valuable aspect to decision-makers.[1] In the bottom line, the freedom of choice literature contradicts some results present in Neary and Roberts (1980), where it is developed the basis of consumer's behavior under rationing, i.e., when at least one of the goods is quantity constrained. In the theoretical framework of these authors, it is not considered the influence of the freedom of choice element, meaning that a rationing situation can generate the same level of "satisfaction" to the consumer observed that the offered bundle under rationing is the same one that he would naturally choose (given his budget constraint) in a non-rationing situation.

To a large extent, this literature criticizes welfare economics based solely on Utilitarianism terms. Sen (1991) points out that Utilitarianism can be factorized into three different aspects: *welfarism*, which denotes the evaluation of any social state exclusively in terms of the utilities generated in this state; *sum ranking*, that sums individuals' utilities to aggregate them, and *consequentialism*, where outcomes are exclusively assessed in terms of this consequences. In this context, many variables that may be pertinent to an appropriate understanding of "welfare", or "social welfare" are not being considered, which is the case of freedom of choice in the present discussion. Hence, the process that leads an individual or a society to reach a certain goal or social state is relevant in the evaluation of this final result.

Thus, the idea that motivates the majority of the freedom of choice literature is that freedom has *intrinsic importance* to human beings. And if this statement is true, when analyzing political elections, not only the possibility of choosing an adequate candidate, or having several alternatives of choice, are valuable to electors, but the process of choice itself can be a source of well-being to individuals in the extend that it can provide larger freedom of choice. Hence, this essay aims to discuss the freedom of choice aspects during

[1] To exemplify, the authors imagine a situation where there is only two goods (x_1 e x_2), and that, after realized the utility maximization process subject to the budget constraint, an individual concludes that the optimal bundle is given by one unit of each good. In this scenario there at, at first, infinite feasible consumption bundles given the budged constraint, and observed all this options is that is given the choice process. Nevertheless, imagine that the individual do not have infinite possibilities of combinations between x_1 e x_2, and for him is offered directly the bundle $x^* = (1, 1)$. In this case, according to the traditional literature, both situations are equivalent to this individual, while the freedom of choice literature would say that the first situation is preferable to the second one. (GAERTNER; XU, 2011, p. 718)

political elections, focusing on the impacts of uncertainty regarding candidates to the degree of freedom experienced by voters, where candidates are seen as lotteries on an issue space, called *ideological space*, following the spatial models of voting with uncertainty (e.g. Shepsle (1972), McKelvey (1980)).

To put forward our analysis, two approaches are used: the simple cardinality-based approach of Pattanaik and Xu (1999), and the known-option-based rule of Arlegi and Dimitrov (2005). The first theoretical formulation do not give any role to the uncertainty aspect, while the latter can be seen as a reformulation of the simple cardinality-based rule when individuals consider the uncertainty about alternatives as a factor that can constrain their freedom to choose. Both rules are compared, and it is suggested an entropy-based rule to classify alternatives as "known" and "unknown"; categories which are needed to rank sets of alternatives in terms of freedom of choice in accordance to the approach of Arlegi and Dimitrov (2005), where we suggest that an alternative will be "known" if it lies within a certain range of "tolerable entropy", which can be translated as "tolerable uncertainty" as well.

Some conclusions are then presented:

1. both approaches yield the same results only if all candidates are considered as "known" by voters, which can be the case if all voters are completely tolerant towards uncertainty;

2. if the competitors are highly ambiguous about their political platforms (Shepsle (1972), Page (1976), Alesina and Cukierman (1990), Glazer (1990), and others) then it is possible to reach an election procedure where all candidates are considered as "unknown", leading to a severe constraint in the freedom of choice experienced by voters during the choice procedure;

3. the rule of Arlegi and Dimitrov (2005) invoking total priority to known candidates may lead to a counterintuitive result, where an election with only one candidate provides more freedom than an election with many candidates, observed that in the former election the only runner is "known", while in the later all candidates are "unknown".

Besides this introduction, this essay is divided as follows: Section 2 presents an overview of freedom of choice approaches that already have been proposed in the literature, while in Section 3 those approaches are discussed and compared, highlighting their relevant aspects. During Section 4, the models of Pattanaik and Xu (1999) and Arlegi and Dimitrov (2005) are detailed within the context of elections, and some discussion about the “known-unknown” problem is also made, where it is also presented the idea of using the entropy of the lottery representations of candidates to classify them into these two labels, and the main results of this essay as well. Finally, some conclusions and ideas for future work are explored in the final section.

2 Measures of Freedom of Choice

Different measures of freedom of choice have been proposed by the literature in the past three decades. As said in Bavetta (2004), most of these measures is rooted in the idea of *positive* and *negative* liberties; concepts that were first introduced by Berlin (1969). Sen (1988) associates negative liberty to “the absence of a class of restraints that one person may exercise over another, or indeed the state may exercise over the individual” (SEN, 1988, p. 272). Thus, while the negative liberty is concerned with the constraints faced by the individual, the concept of positive liberty is related to the possibility of acting. For Berlin (1969), the positive sense of liberty

> [...] derives from the wish on the part of the individual to be his own master. I wish my life and decisions to depend on myself, not on external forces of whatever kind. I wish to be the instrument of my own, not of other men’s acts of will. I wish to be a subject, not an object; to be moved by reasons, by conscious purposes, which are my own, not by causes which affect me, as it were, from outside. I wish to be somebody, not nobody; a doer - deciding, not being decided for, self-directed and not acted upon by external nature or by other men as if I were a thing, or an animal, or a slave incapable of playing a human role, that is, of conceiving goals and policies of my own and realizing them. [...] (BERLIN, 1969, 131)

One approach that takes into account opportunities as a signal of freedom, and stated the formal basis to an axiomatic evaluation of preferences that take into account freedom of choice as an relevant aspect is the *simple cardinality-based* approach, given in Pattanaik and Xu (1990), where it is adopted a pure quantitative view of freedom, i.e., the greater is the number of available options to the individual, the wider is the freedom that he can experience. Formally, suppose that there are two sets of alternatives, A and B; if the set A possesses more elements than the set B (that is, the cardinality of A is greater than the cardinality of B), then it is possible to conclude that A is preferable to B in terms of freedom.

Despite the evident importance of this theoretical formulation to the discussion of the role of freedom and its measurement, one cannot avoid noticing that the cardinal approach is rather simple - as said by the authors themselves, “the simple cardinality-based rule is a rather naive or trivial rule for judging the degree of freedom of choice”

(PATTANAIK; XU, 1990, p. 389) - since it does not take into account some elements that may affect peoples' perception about opportunities, such as the degree of similarity among the options in a choice set.

To illustrate this point, the authors give the following example: assume that distinct modes of transport are being evaluated, e.g. {train} and {blue car}. If freedom of choice is interpreted in accordance to the simple cardinality-based rule, we have that the individual must be indifferent between both sets, observed that either have the same number of options, meaning that they provide the same degree of freedom. Now, suppose that the option sets are given by {train, red car} and {blue car, red car}. Again, since both sets have the same cardinality, by this axiomatic formulation the individual must be indifferent between both option sets. However, it is plausible to imagine that the individual feels the set {train, red car} as a more "freedom-providing" set than the {blue car, red car} set, even observed that they have the same cardinality, since the former provides more diversity to the decision-maker. Hence, if that is the case, the simple cardinality-based approach cannot describe accurately peoples' feelings toward liberty, and there are other factors that must be considered in order to reach a proper theory.

In another paper, Pattanaik and Xu (2000) refine their simple cardinality-based approach to incorporate the similarity between alternatives on an opportunity set as a relevant factor to judge in which extent this set provides freedom of choice. This new theoretical formulation was named as *simple similarity-based* ordering of sets in terms of freedom. In the axiomatic characterization proposed by these authors, adding new alternatives to the opportunity set that are similar to the already existing ones do not increase the degree of freedom experienced by the decision maker.

To illustrate, they present the following example: if a set of alternatives given by {red bus} is enlarged by the option {blue bus}, then it would be plausible to imagine that the resulting {red bus, blue bus} set offers the same degree of freedom that the one provided by the {red bus} set, observed the similarity between traveling by a red or a blue bus; but if instead of {blue bus} the option {red train} appears, then, given the dissimilarity between the new and the previous existent alternative, we can expect the set {red bus, red train} to provide a larger amount of freedom than the one provided by the {red bus} set. Nevertheless, there are no considerations about the intensity of the similarity and dissimilarity between objects, i.e., their axiomatic characterization only classifies alternatives as "similar" or "dissimilar", without exploring how much (dis)similarity exists among options. Moreover, one must inquire how is defined similarity within this context. According to the authors, similarity of options is a matter of social judgment or norms, implying a "non-personalistic" view of similarity; that is, even if the decision maker judges {red bus} and {blue bus} as totally distinct options, the social norm that judges both options similar will prevail upon this individual's perception.

The notion of diversity and similarity, and their relations with freedom of choice, are also examined in Bervoets and Nicolas (2007), where their diversity ranking is constructed in order to allow more than two possible classifications of similarity. In their formulation, to state that a set A is more dissimilar than a set B, it would be necessary to compare the dissimilarity of A's most dissimilar alternatives to B's most dissimilar alternatives; additionally, to say that one set is more diverse than another would amount to say that this set provides more freedom of choice as well (BERVOETS; NICOLAS, 2007, p. 268).

In order to clarify this perception of freedom, the authors use another transportation example, where the set of conceivable transportation modes between two cities is given by $\{$bike, car, foot, train$\}$. If we imagine that there are more differences between taking a train and walking than, say, between any other pair of alternatives that can be formed by the elements of the set $\{$bike, car, train$\}$, then it is plausible to conclude that the subset $\{$foot, train$\}$ is more dissimilar than the set $\{$bike, car, train$\}$. However, it is worth noting that, focusing on the most dissimilar objects, this criterion would also state that the set $\{$foot, train$\}$ provides the same diversity degree as the one given by the set $\{$bike, car, foot, train$\}$, what seems counterintuitive at first glance since the latter set offers more alternatives to the decision maker than the former.

Besides the importance of diversity to freedom of choice, not much have been said in these models about the decision maker's preferences over the alternatives on the opportunity set. Sen (1988, 1993) broadens the freedom concept incorporating individuals' preferences as a relevant element to the measurement of freedom. According to him,

> the evaluation of the freedom I enjoy from a certain menu of achievements must depend to a crucial extent on how I value the elements included in that menu. The 'size' of a set, or the 'extent' of freedom enjoyed by a person, cannot, except in very special cases, be judged without reference to the person's value and preferences. (SEN, 1993, p. 528)

Thus, according to Sen, the simple cardinality-based, or the simple-similarity based approaches would be feasible only in "very special cases", when the individual feels that there are no qualitative considerations to be made about the presented options. Thus, Sen suggests that qualitative factors play an important role in determining freedom. To illustrate how important is the quality factor in the analytical process of freedom of choice, Sen states that a pure cardinal view would lead people to accept that "three alternative achievements that are seen as 'bad', 'terrible', and 'disastrous' gives us exactly as much freedom as a choice over another three alternative achievements which are seen as 'good', 'terrific' and 'wonderful' " (SEN, 1993, p. 529). In other words, besides the number of alternatives that an agent can access, any measure of liberty must also look at how this agent ranks these available alternatives considering his preferences among them.

Peragine and Romero-Medina (2006) elaborate a model where both diversity of options and preferences over them are significant factors to measure the freedom of choice that an individual experiences. The authors use the binary notion of similarity present in Pattanaik and Xu (2000) and introduce individual's preferences over these alternatives as an additional information to create two possible ranking rules in terms of freedom. In the first ranking, priority is given to the opportunity aspect, characterizing the relation between freedom, diversity and preferences in a cardinal fashion, while the second ranking gives priority to the diversity aspect. In other words, in the first ranking it is imagined that the individual first selects the options that are considered as the most relevant (given all feasible preferences over them) and then count them disregarding similar options as different options, while in the second ranking the first step would be filtering options using as a parameter the diversity among them, to then focus on the remaining relevant alternatives in terms of the individual's preferences.

Other relation that has been object of investigation is between freedom, uncertainty and entropy. Some authors, as Suppes (1996) and Erlander (2010), relate the freedom of choice provided by a procedure to its effective results. Analyzing elections outcomes, for example, Suppes (1996) suggests that the greatest freedom of election would be achieved when the number of candidates is high and each one receives almost the same number of votes. As said by the author, "it would be surprising to have a high measure of freedom for the process and a low one for the result" (SUPPES, 1996, p. 188). Moreover, the entropy can be used to measure the diversity of opportunities as well. To exemplify, the author resorts to the following example: suppose that there are two candidates and m relevant characteristics that are relevant for a candidate in an election. Consequently, there are $t = 2^m$ possible types of candidates given those m characteristics, where each candidate can assume one of this t types during an election campaign. Hence, if both candidates assume the same type, the entropy of this election is zero, culminating in a sharp decrease of freedom of choice.

Jones and Sudgen (1982) explore the concept of *significant choice*. In their conceptual framework, a choice set would offer significant choices to the extent that it helps the development of intellectual and moral aspects of the individual, and provides a wide diversity of alternatives. Then, one sort of choice that is not significant in this sense is when the decision maker do not have any reason for choosing a specific option instead of choosing any other available alternative. As argued by the authors, the decision maker "knows immediately that he might as well choose at random, and has no need to tax his mental faculties any further" (JONES; SUDGEN, 1982, p. 59). The concept of significant choice is linked with freedom by the idea of *autonomous choice*. A decision maker is autonomous when he consciously decides which option is the most valuable to him, which can be loosely stated as using his mental faculties to put forward the decision process. In the voter decision problem, it is plausible to imagine that the lack of relevant information affects his autonomy, since there is no reasonable judgment to be made that can help him to effectively access a decision that reflects his real preferences among the candidates. As Jones and Sudgen (1982) argue, "it is not simply that the chooser is indifferent between the two options: one cannot conceive of a reasonable person in the position of the chooser being anything but indifferent". (JONES; SUDGEN, 1982, p. 59)

The same concept of "reasonable person" and its relations to autonomous choice was addressed in Pattanaik and Xu (1998), where it is considered an *autonomous agent* the individual that chooses on basis of his preference pattern, but that could easily have chosen differently using as parameter another perfectly reasonable preference pattern that he could eventually hold. The authors give an interesting example to illustrate how a "reasonable person" concept may (or may not) work: imagine an opportunity set that is expanded by adding the option "beheaded at dawn", then, it is feasible that the freedom experienced by an agent facing this set is not increased at all, even observed that this expanded set has more alternatives than the previous one. The reason do not depend much on the fact that this individual probably prefers the already existing options than the new one, but mainly it "lies in our presumption that, given the circumstances of the agent, no *reasonable* person would prefer the option of being beheaded at dawn over the other options" (PATTANAIK; XU, 1998, p. 179). Nevertheless, suppose now that the former set was given solely by the option "spend the rest of his life in a solitary cell" (which could be, for instance, 50 years); then, the authors say that adding the alternative "beheaded at dawn" to this opportunity set may now increase freedom of choice, since one can reasonably prefers being beheaded at dawn to spent 50 years in a solitary cell. (PATTANAIK; XU, 1998, p. 179)

Another interesting feature that can be used in the freedom of choice analysis is the concept of *reference point*. The idea of reference point is well explored in the psychology and economics literature; as Rabin (1998) points out, there are several studies that show that individuals are more sensitive to how their actual situation diverge in relation to a certain "reference point", than in relation to absolute characteristics of that situation. A typical example is the temperature: the same temperature that feels cold when one is adapted to the heat can seem hot when one is adapted to the cold (RABIN, 1998, p. 13). Gaertner and Xu (2011) develop a model where individuals can evaluate each opportunity set in terms of the freedom that they offer using as a parameter a reference point, which symbolizes an option that permits a "minimum level of achievements", where any alternative that do not allows at least the achievement of this minimum level will make the individual's life unpleasant or miserable (GAERTNER; XU, 2011, p. 718). In their axiomatic formulation, to a certain set of options A be preferable to another set B - in the freedom of choice sense - it is not only necessary the cardinality of A to be greater than the cardinality of B, or that A, in absolute terms, offers better options than B; it is also necessary that, among A's options, exist one that is considered the "reference point", and that in terms of this point the options of A are judged more valuable than the available options of B.

So far, all freedom approaches already presented do not incorporate explicitly uncertainty or the lack of information as relevant elements to the analysis of freedom of choice. Arlegi and Dimitrov (2005) propose an axiomatic approach to represent freedom of choice under the influence of this informational constraint. First, it is distinguished between the alternatives that are "known" from the ones considered "unknown", where "known" options are those "whose relevant characteristics in order to be evaluated and compared with other options are known by the agent" (ARLEGI; DIMITROV, 2005, p. 4), while the "unknown" alternatives are the remaining ones. The authors suggest that incorporating an "unknown" option into the opportunity set may not increase the freedom of choice enjoyed by this individual, justifying this sort of behavior through the existence of a preference for *easy choices*, i.e., since it would be hard to define precisely the characteristics of an unknown alternative, decision under such circumstances would become a difficult task to the individual, culminating in this kind of aversion to alternatives whose relevant characteristics are not well stated. Hence, freedom of choice is enhanced to individuals only if "known" alternatives are added to the opportunity set.

3 Freedom and Elections

Many different conclusions can be reached about freedom of choice in political elections, depending on which approach of freedom one relies. As already said, our main focus during this work will be the approaches of Pattanaik and Xu (1990), and Arlegi and Dimitrov (2005), that will be explicitly studied in the next section, but it seems interesting to discuss how the other proposals for measuring freedom can be convenient, and which points remain uncovered by them. To illustrate, imagine a presidential election with $N > 0$ distinct candidates, i.e., there are N different persons that are running in this election. As a starting point, let us assume the Pattanaik and Xu (1990) simple cardinality-based approach. Then, there is little to say about preferences among candidates, uncertainty, autonomy or any other factor, leading us to conclude that this election provides more freedom of choice than any hypothetical election with, say, $M < N$ competitors. Hence, enlarging N would be equivalent to enhance freedom.

If we assume the point of view adopted in Sen (1988, 1993), or if the reference point concept of Gaertner and Xu (2011) is used, then some considerations about how agents perceive the N options must be made. Imagine that one "left-wing" voter, after analyzing the profile of the N available candidates, judges them as "right-wing" candidates. Then, even if there is one candidate which is closer to a "leftist" ideological position, the degree of freedom experienced by this voter will be very low, since all competitors are seen as bad options given that no one can represent this voter's ideological opinion, or, put in other terms, the reference point of this voter is not an element of the options' set. In such a situation, comparing this N ideologically equal candidates' election to another hypothetical one with $M < N$ competitors, but where at least one of the M candidates represents his most preferred ideological position, would probably lead this voter to the conclusion that the M candidates' election provides more freedom of choice than the other with more competitors. Some similar conclusion can be reached if diversity is taken into account, that is, an election with fewer candidates can perform better in terms of freedom if all competitors are ideologically distinct, meaning that improvements in freedom are only possible when are added new distinct options - in relation to the previous existent ones - to the set of candidates.

All notions of freedom but simple cardinality-based need some sort of judgment of alternatives in terms of how similar they appear to be, or in terms of their potential quality. Then, incorporating uncertainty into the decision process, as well incomplete information and other information processing problems, seems to be justifiable. In Tversky and Kahneman (1974) it is explored the idea that individuals, when face decision problems under uncertainty and informational constraints, use a limited number of *heuristic principles* to put forward the judgmental operations that are needed in the decision process, which can lead the individual to severe errors of judgment. Back to the election example, this means that the existence of a similarity problem among the N available candidates may not be due to "real" similarity, i.e., each of them really having the same ideological position; instead, the origins of these problems may be rooted in how poor informed voters are to judge precisely which are the ideological positions of the candidates.

The implications of this poor judgments depend on which idea of freedom one relies: if a "left-wing" voter concludes that all N candidates are also "leftists", and his interpretation of freedom relates to Sen's approach, or the reference point idea of Gaertner and Xu(2011), then he will probably feel that his freedom of choice is being restrained, but certainly in a slighter intensity than if compared to a situation where all candidates were judged as "right-wing". On the other hand, if this "left-wing" have considered all N

candidates as “rightists”, and assuming this left-wing ideological position as a reference point - that is, the minimum level of achievements acceptable - then this impossibility of choosing a candidate that has an attractive ideological position as him not only diminishes his freedom, but also can make the decision process something “painful”.[2]

Notwithstanding, both situations would be indistinguishable if freedom were associated only to the diversity of alternatives that an option set offers. That is, using the simple similarity-based approach of Pattanaik and Xu (2000), since the diversity given by a set of only “right-wing” candidates is the same observed in a “left-wing” candidates set, then both sets should provide the same freedom to an individual, regardless this individual’s political preferences.

Considering the autonomy approach of Jones and Sudgen (1982), and Pattanaik and Xu (1998), as one pertinent factor for freedom of choice, then one can become skeptical about Suppes (1996) approach, where the result of the election can (with a reliable degree of accuracy) measure of the freedom that voters experienced during the choice procedure. We suggest the following example in order to clarify this point. Suppose that there are two candidates, called A and B, and N voters, each one indexed by $i = 1, \ldots, N$, with N being a large number.[3] Also, assume that, during the electoral campaign, voters do not enjoyed any relevant information that would help to conclude that one candidate is better than others. Then, it is feasible to imagine that voters will be indifferent between these candidates, since they do not have any reason to believe that one is better than the other, leading them to choose randomly which candidate would receive the vote. Imagine that the voting procedure, for every $i = 1, \ldots, N$, is the following: the voter tosses a fair coin and observes the side that appears; if it is heads, he votes for A, but if is tails, he votes for B. Consequently, the expected share of votes for A and for B is 50%, which also implies in the larger entropy measure and the wider degree of liberty as well.

But can we say that these voters *experienced* real freedom when decided to vote for A or B? Would not they prefer to conscientiously vote for A (or B), after a deep analysis of the relevant characteristics, such as historical background and the political platform that is being purposed, instead of choosing this candidate using such a naive procedure? Or, put in another words, do voters fell that their choices are *significant*, in the sense of Jones and Sudgen (1982), in such an environment? If one has in mind the relation between

[2]The german word *weltschmerz* can help in the understanding of this idea. This term, created by the writer Jean Paul Richter between the XVIII and XIX centuries, can be comprehended as the depression that one experiences when compares the real world to a hypothetical idealized world, realizing that this real world do not - and maybe never will - fulfill his idealized world characteristics. In this example it would be the felling that the voter experiences when he recognizes that his ideal world (the reference point, his left-wing ideological position) is inviable given that no available candidate represents it.

[3]This example could typically describe the second round of a runoff voting to elect a president, for instance.

freedom and autonomy, we could arguably conclude that the final expected outcome of 50% of votes to each candidate will have little to say about freedom.

The argument of autonomy, however, can provide some counterintuitive results. Let us use the "beheaded at dawn" example of Pattanaik and Xu (1998), adapting it to our discussion about freedom of choice during elections. Suppose that there is an already known number of candidates running for president, and that this existent set is enlarged by adding a new option, that is considered as "undesirable" by voters. Hence, it is plausible to expected a little contribution of this alternative to voters' freedom of choice, since no reasonable person would vote for someone who has "undesirable" as a main adjective. However, imagine that all other alternatives were judged as, say, "incredibly undesirable". In this case, the same conclusion reached on Pattanaik and Xu's example is valid here: voting for the less "undesirable" alternative, among all bad options that are unfortunately available, seems the only reasonable thing to do. Thus, since we are adding a *reasonable* alternative to the opportunity set, we may also expect freedom of choice to be increased. But what if the original candidates were judged as "excellent" alternatives, while the new one were considered as " incredible excellent"? If that is the case, then one can expect voters to have many reasonable voting options at first, i.e, they could vote for any specific "excellent" candidate, at the same time that they could reasonably choose any other of the "excellent" group. Adding an "incredible excellent" option to the opportunity set, however, would presumably reduce the number of reasonable choices to just *one* - this new "incredible excellent" candidate - since voting for "excellent" instead of "incredibly excellent", by this qualitative logic, would be an unreasonable behavior.

After this discussion, we may now present the two approaches that are going to be described in more details during this essay: the *simple cardinality-based* approach of Pattanaik and Xu (2000), and the *known-options-based* rule of Arlegi and Dimitrov (2005).

4 Models

First, let us expose the basic notation that will be used during this section, following the works of Pattanaik and Xu (1990), and Arlegi and Dimitrov (2005). Denote by I the set of individuals in this society, where each individual is indexed by $i \in \{1, \ldots, \#I\}$. Each individual is assumed to vote, but may also be a candidate running in the election. Hence, every individual is a potential alternative for office.

Let Z be the set of all non-empty subsets of $i \in I$, i.e., the set of all possible subsets of individuals that can be formed with $\#I$ agents, and define $Z^* = Z \cup \{\varnothing\}$. Note that $I \in Z$ as well. An election is a set $M \in Z$ of voters that are also candidates in a certain period of time.[4] Hence, the set Z is interpreted here as the set of all possible elections that can be formed with $\#I$ voters, with cardinality equal to $2^{\#I} - 1$. Voters and candidates are distinguished in the following sense: an individual i such that $i \in I$, but $i \notin M \cap I$, will be a *voter*, and still will be denoted by $i \in I$, while an individual i such that $i \in I$, and $i \in M \cap I$, will be a *candidate*, and then denoted by $m \in I$. Denote by $K_i \in Z$ the set of individuals that are "known" to the specific individual $i \in I$, and by define the set Θ^i_m as the set of *relevant* information that voter i has gathered about candidate a $m \in M$.

Following the tradition of spatial models of voting (Shepsle (1972), Enelow and Hinich (1982, 1984)) it is assumed that individuals vote ideologically, and denote by $X = \{x_1, x_2, \ldots, x_N\} \in \mathbb{R}$ a finite set of K distinct ideological positions that individuals (voters and candidates) can assume.[5] Define $\mathbb{P}$ as the set of all possible finite lotteries over X, where, for all $i \in I$ and $m \in M$, $\mathcal{P}^i_m \in \mathbb{P}$ denotes the lottery (i.e., the set of probabilities) that express i's beliefs towards candidate m's ideological position on X. Additionally, define $\mathbb{P}^i_M \subset \mathbb{P}$ as the set of lotteries used to describe candidates for the i voter, and, using the concept presented in Shannon (1948), $H(\mathcal{P}^i_m)$ as the *entropy* of the probability distribution $\mathcal{P}^i_m$.

4.1 The Simple Cardinality-Based Approach

Let us follow Pattanaik and Xu (1990) and analyze freedom of choice on political elections within a purely cardinal view. Assume that $\succsim^Z_i$ is a binary relation over Z, that is, over all possible elections that can be formed with $\#I$ individuals, where, for any $M, M' \in Z$, $M \succsim^Z_i M'$ may be read as "the feasible election M offers at least as much freedom of choice to the voter i than the feasible election M'", while $M \succ^Z_i M'$ and $M \sim^Z_i M'$ denote the asymmetric and symmetric relations, respectively. In order to describe preferences that only take into account the cardinal element, the authors introduce the following axioms.

Axiom INS (*Indifference Between No-choice Situation*)**.** For all $m, m' \in M$, $\{m\} \sim^Z_i \{m'\}$

Axiom SM (*Strict Monotonicity*)**.** For all $m, m' \in M$, with $m \neq m'$, $\{m, m'\} \succ^Z_i \{m\}$.

Axiom IND (*Independence*)**.** If, for all $M, M' \in Z$, and $m \in Z - (M \cup M')$, $[M \succsim^Z_i M'$ iff $M \cup \{m\} \succsim^Z_i M' \cup \{m\}]$.

[4] Thus, an election here symbolizes the set of alternatives presented to individuals, and not the voting rule that is being used.

[5] Some models, such as Anderson and Gloom (1991) and Shepsle (1972), assume that the "ideological spectrum", or the "policy space", are a continuum. However, it is not possible to affirm that the continuum assumption have more desirable features than the discrete case. Moreover, if one assumes that K is a sufficient large number, then it possible to approximate the discrete to the continuum case.

Definition 1 (*Simple Cardinality Ordering*)**.** For all $M, M' \in Z$, $M \succsim_i^Z M'$ iff $\#M \geq \#M'$.

Each of these properties have interesting implications on individuals' understanding of freedom of choice. If we assume a pure quantitative view of freedom, then the INS axiom seems intuitively plausible. For instance, suppose two possible electoral scenarios: in the first one, the candidate $m \in I$ is the only competitor, what culminates in $M = \{m\}$, while in the second, another individual $m' \in I$, with $m' \neq m$, is the unique candidate, defining the election $M' = \{m'\}$. In this case, since it is not being given any role to preferences, uncertainty or any sort of qualitative weight to alternatives, voters must not encounter differences in terms of freedom in both elections, no matter how "good" or "bad" a voter may feel m in relation to m'.

The strict monotonicity property also have strong appeal. As the authors argue, the SM axiom "embodies the principle that, in terms of freedom, a situation where the agent has some choice is better than a situation where the agent has no choice" (PATTANAIK; XU, 1990, p. 387), or, put in other words, the greater the number of alternatives, the larger is the degree of freedom. Hence, an election $M = \{m, m'\}$ will be better, in terms of freedom, than another election $M = \{m'\}$.

Finally, the IND axiom states that, given two possible elections M or M', incorporating a candidate m to both elections must not change how individuals perceive freedom in M or M'. For instance, a voter cannot fell M as a better election than M' in terms of freedom, but at the same time believe that $M' \cup m$ provides greater freedom than $M \cup m$.[6]

The main result of Pattanaik and Xu is given in the following theorem.

Theorem 1 (*Pattanaik and Xu, 1990*)**.** $\succsim_i^Z$ is the Simple Cardinality-Based Ordering if and only if $\succsim_i^Z$ satisfies INS, SM and IND axioms.

The Simple Cardinality-based Ordering stated the foundations for a vast literature on axiomatic approaches of freedom. Introducing a relatively simple set of axioms, they describe a rule that only needs to evaluate the number of alternatives in each set in order to compare which one provides greatest freedom.

4.2 The Known-Options-Based Rule

In this section we present the approach of Arlegi and Dimitrov (2005). This theoretical formulation is interesting to our purposes since it establishes a simple criterion to determine the role of information to freedom during choice processes, allowing a direct

[6]Provided that m is not a candidate in both M and M'.

application of the entropy concept to the measurement of this freedom, as we suggest latter. Using their axiomatic formulation, freedom of choice would be increased in political elections only if "known" candidates are added to the opportunity set. In order to be considered as a "known" option, the individual must have enough *relevant information* about the alternative, where a piece of information is considered relevant if it helps the agent during the choice procedure.

Freedom of choice is measured comparing *extended opportunity sets*, which are given by $(M, K_i) \in Z^* \times Z$.[7] Contextualizing into the political elections scenario, an extended opportunity set is given by a set of effective options (that is, candidates) denoted by the election M, upon which a decision must be made, and a set of known options (i.e., voters and candidates that the individual i considers as "know") K_i that will help this individual to perform his decisions over M. For notation simplicity, define $\mathcal{Z} = Z^* \times Z$, where $\succsim_i^{\mathcal{Z}}$ denotes a complete and transitive binary relation on $\mathcal{Z}$. To formalize, they suggest the following set of axioms.

Axiom EC (*Empty Choice*)**.** For all $K, K' \in Z$, $(\varnothing, K) \sim_i^{\mathcal{Z}} (\varnothing, K')$

Axiom SM* (*Simple Monotonicity*)**.** For all $m \in M$, and $m' \in K \in Z$, $(\{m, m'\}, K) \succ_i^{\mathcal{Z}} (\{m\}, K)$.

Axiom SN (*Simple Neutrality*)**.** For all $m \in M$, and $m' \notin K \in Z$, $(\{m, m'\}, K) \sim_i^{\mathcal{Z}} (\{m\}, K)$.

Axiom IND* (*Independence*)**.** For all $(M, K), (M', K') \in \mathcal{Z}$, and for all $m \in I \backslash M, m' \in I \backslash M'$, with $m \in K \Leftrightarrow m' \in K'$, then $[(M, K) \succsim_i^{\mathcal{Z}} (M', K')$ iff $M \cup \{m\} \succsim_i^{\mathcal{Z}} M' \cup \{m'\}$.

Definition 2 (*Known-Options-Based Rule*)**.** For all $(M, K), (M', K') \in \mathcal{Z}$, $(M, K) \succsim_i^{\mathcal{Z}} (M', K')$ iff $\#(M \cap K) \geq \#(M' \cap K')$.

The main difference between this ordering and the simple cardinality-based approach is due to SM and SN axioms. While SM axiom in Pattanaik and Xu (1990) states that freedom is always increased no matter which sort of alternative is added to the opportunity set, in the known-option-based approach the decision-maker's freedom is conditional to an informational factor, that is, if an additional alternative is considered as "known", then the simple monotonicity axiom states that the freedom of choice provided by this set will be increased. Conversely, if this new option is classified as "unknown", then, by the SN axiom, there are no changes in the decision-maker's perception of freedom. In what concern the others axioms, the EC axiom states that individuals must be indifferent

[7] Notice that the authors here consider the empty set as a possible opportunity set.

between two empty opportunity sets, whatever is the known set of options (however, since we are assuming that elections must have at least one competing candidate, this property will be of little interest here), while the IND axiom follows the same logic presented in Pattanaik and Xu (1990).

Given the above definitions, Arlegi and Dimitrov (2005) state the following theorem.

Theorem 2 (*Arlegi and Dimitrov, 2005*)**.** $\succsim_i^Z$ is the Known-Option-Based Rule if and only if $\succsim_i^Z$ satisfies EC, SM*, SN and IND axioms.

4.3 The "Known-Unknown" Problem: A Proposal

In order to reach a more precise understanding of freedom of choice during electoral periods within this theoretical framework, it is important to examine how candidates become "known" or "unknown" (what is closely related to the idea of relevant information to voters' decisions). So, if electors only need to know candidates' parties to reach a coherent decision, and assumed that each candidate cannot run for office without being linked to a political party, then, since this piece of information is presumably available to voters without any sort of restriction, one can expect the known-option-based rule to perform the same results presented by the simple-cardinality approach. In this case, each new candidate will inevitably increase freedom of choice, and an election will reach the maximum "degree of freedom" in the unlikely scenario of each voter becoming a candidate. Nonetheless, if more information is needed to reach a considered decision, and, more importantly, information that is not directly observable, or collected so easily as candidates' parties, then the known-option-based rule may lead to the conclusion that freedom of choice is severely constrained by the lack of a solid basis of relevant information, despite the number of running candidates.

Assuming that individuals vote according to their ideological preferences, and that each voter represents candidates by lotteries over a theoretical "ideological space", could be a useful manner to deal with the "known-unknown" problem. Let us assume that the only relevant information to the voting decision is the ideological position that each candidate assumes, and that voters are uncertain about this information, leading to lotteries representations in the ideological space. Since we are supposing the existence of these probabilistic representations of candidates - which are subjectively defined considering any informational basis available - then one can expect candidates to be considered as "known" only when their respective lotteries representations are degenerated, that is, to a "known candidate", voters would assign a probability 1 for a certain outcome in X, and 0 to the others. However, this would probably lead our analysis toward a trivial result, when all candidates are considered as "unknown" by definition. Then, we suggest here that there is a certain "amount" of uncertainty, personified by a lack of relevant information, that voters tolerate, meaning that candidates can be "known" even if their lottery representation is not degenerate on the ideological space.

We now introduce our first assumption about the "known-unknown" classification.

Axiom TE (*Tolerable Entropy*). For all $i \in I$, $m \in M$, and $K \in Z$, $m \in (M \cap K) \Leftrightarrow H(\mathcal{P}^i_m) \leq \tau_i$.

In words: voter i feels that he knows a specific candidate m when the entropy of the probability distribution used to represent this candidate on the set X is lower than a certain level of "tolerable" entropy, or uncertainty regarding candidate's ideological position. By the properties of the entropy function, this level of uncertainty must lie between 0 and the maximum entropy achievable by a probability distribution in the set X, which is associated to an uniform $\mathcal{P}^i_m$.

It is important to note that no considerations are being made about how accurate are voters' evaluations, or if "known" candidates are those whose *real* ideological preferences are effectively known. For instance, let us say that there are two ideologies, L and R, upon which voters evaluate candidates. Then, it is possible to imagine a situation where one voter feels that a candidate m is "known", attributing to him a the L ideology, while other voter also considers this candidate as "known", but assigning to him the R ideology. Since this candidate must be "left", or "right", but cannot be both at the same time, then one of these voters has wrong beliefs about this competitor ideology, regardless the fact that he is sure about them.

5 Results

First, let us use the simple-cardinality based approach, where the following proposition becomes obvious.

Proposition 1. If, for all $i \in I$, $\succsim^Z_i$ is the Simple Cardinality-Based ordering, then an election $M \in Z$ such that $M \subseteq I$ and $I \subset M$ is the one that provides the highest degree of freedom of choice.

Proof. Suppose an election $M' \in Z$, with $\#M' = n$, where n is any natural number of the interval $0 < n < \#I$. By definition, we know that $\nexists\ m \in M'$, but $m \notin I$, which means that there are $\#I - n$ individuals $i \in I$ that are not in election M'. Let the cardinality of the set $I \backslash M'$ be $\#I - n$. Hence, $M' \subseteq I$, but $I \nsubseteq M$. Take an alternative election $M'' \in Z$ and a nonempty subset of individuals $A \subset (I \backslash M')$ such that $M'' = M' \cup \{A\}$. Clearly, $\#M'' = n + \#A > \#M'$, implying in $M'' \succ^Z_i M'$.

For clarity, let us write $\#I = n + \#I - n$. Then, since $\#A < \#I - n$, we have that $\#M'' = n + \#A < n + \#I - n$, and hence there are $\#I - n - \#A$ individuals that are not in election M'', which means that the cardinality of the set $I \backslash M''$ is $\#I - n - \#A$, and $M'' \subseteq I$, but $I \nsubseteq M''$.

Assume now that $A' = I \backslash M''$, and that there is an election $M \in Z$ such that $M = M'' \cup A'$. Then, $\#M = \#M'' + \#A'$, which can be written as $\#M = n + \#A + (\#I - n - \#A)$, that finally yields $\#M = \#I$. Since, for any n in the interval $0 < n < \#I$, we have $\#M > \#M'' > \#M'$, then $M \succ_i^Z M'' \succ_i^Z M'$, i.e., M provides more freedom of choice than any other election with cardinality less than $\#I$. Also, observed that $\nexists\, m \in M'$, but $m \notin I$, then $I \backslash M = \varnothing$. Thus, $M \subseteq I$ and $I \subseteq M$, and there is no subset of individuals in the set I that can be added to the set M, meaning that $\nexists\, M \in Z$ with $\#M > \#I$, and that M provides more freedom than any other election in the set Z. □

In words: the Proposition 1 above states that the most freedom-providing election would be the one where each individual postulates his candidacy for office. This result emerges since any additional factors are being considered than the cardinal one. That is, the cardinal argument do not take into account preferences over candidates and their opinions on issues, the potential uncertainty that can arise regarding the characteristics of those candidates, or even the similarity among options. In the simple-cardinality based approach, increasing the number of alternatives not only increases freedom of choice, but also does it at a "constant rate", i.e., every new candidate enhances freedom by the same amount enhanced by the previous candidate that entered the competition.

Uncertainty would only affect the perception of freedom if voters were not sure about how many alternatives they have for choice, or if they do not have this information. For instance, imagine that, when comparing two elections $M, M' \in Z$, voter i is informed that M has 5 candidates, while the number of candidates in M' is an element of the set $\{4, 5, 6\}$. If this elector perceives freedom of choice according to the simple cardinality-based rule, then he knows that M' can provide greater, equal or less freedom in comparison to M (depending if the number of candidates is, respectively, 6, 5 or 4), but does not know which one is the real case. Note that uncertainty in this case does not arise as a consequence of ambiguous relations between candidates and their position on relevant issues, but it is related solely to the cardinality of the sets that are being compared, no matter which are the alternatives that they effectively present.

The authors themselves suggest that this approach is not convenient for measuring freedom in many different situation. But measuring freedom only on basis of the cardinality of the opportunity set has an obvious empirical advantage, since the number of candidates is easily observable, while voters' preferences, uncertainty regarding alternatives and the majority of the other factors that have been suggest in the literature demand much more effort to be gathered, or are impossible to be measured.

Let us analyze the effects of uncertainty about alternatives using the known-option-based rule, already considering our suggestion of an entropy-based classification of "known" and "unknown" on freedom of choice. Denote by $\mathcal{X}_m^i$ a random variable that, for elector i, denote the ideological position of candidate m. Assuming that individuals assign probabilities in accordance to the principle of *maximum entropy* of Jaynes (1957, 1968), then probabilities of m's ideological position over X are given by

$$\mathcal{P}_m^i(\mathcal{X}_m^i = x_k \mid \Theta_m^i) = \frac{e^{-\sum_{r=1}^{R} \lambda_r f_r(x^k;\theta_m)}}{\sum_{k=1}^{N} e^{-\sum_{r=1}^{R} \lambda_r f_r(x_k;\theta_m)}} \tag{1}$$

where $\mathcal{P}_m^i(\mathcal{X}_m^i = x_k \mid \Theta_m^i)$ also considers the amount of relevant information that elector i has gathered about this candidate ideological position.

Proposition 2. If, for all $i \in I$, $m \in M$, and $K \in Z$, we have that $\tau_i < \log N$, and $\Theta_m^i = \varnothing$, then $(M \cap K) = \varnothing$.

Proof. It is known that the maximum entropy level occurs when the probability distribution is uniform, and given by $\log N$.[8] Given that the maximum entropy principle is used to assign probabilities, $\Theta_m^i = \varnothing$ implies in a lottery over the set X that is uniform. If, for all $i \in I$, $\tau_i < \log N$, then $\nexists\, m \in M$ with $H(\mathcal{P}_m^i) < \tau_i$, which, by Axiom TE implies in $(M \cap K) = \varnothing$. □

Proposition 2 states that the absence of relevant information about candidates lead to an election without any "known" candidate. One first implication of this situation to freedom of choice is given in the following proposition.

Proposition 3. If $\succsim_i^{\mathcal{Z}}$ is the Known-Option-Based ordering, and $(M \cap K) = \varnothing$, then, for all $M', K' \in Z$, such that $(M' \cap K') = \varnothing$, we have $(M, K) \sim_i^{\mathcal{Z}} (M', K')$.

Proof. This proof follows directly from the definition of the Known-Option-Based rule. Since $(M \cap K) = \varnothing$, and $(M' \cap K') = \varnothing$, then $\#(M \cap K) = \#(M' \cap K')$, and hence $(M, K) \sim_i^{\mathcal{Z}} (M', K')$. □

The above proposition says that individuals are indifferent between any possible non-empty election with only "unknown" candidates. In other words, the cardinality of the opportunity set does not play any role on individuals' perception of freedom when uncertainty regarding their ideological positions are above the tolerable limit.

[8] To a formal proof of this result, see Cover and Thomas (1991).

One counterintuitive implication of this result is that freedom of choice of two elections, one with a single "unknown" candidate, and another with many "unknown" competitors, will be the same. That is, given that freedom is enhanced only when new alternatives can be properly evaluated observed the available information, and that the absence of relevant information imposes a severe curtailment of the capacity of evaluating candidates with a certain degree of accuracy, one must note that an electoral system that is only concerned with providing a large number of candidates will not be an ideal system to enlarge freedom of choice, unless this system also gives conditions for candidates to spread their campaign information properly, reducing voters' uncertainty about the relevant issues to the decision-making process.

Nonetheless, spreading relevant information, i.e., revealing which are their real opinions on issues of campaign, may not be an interesting strategy for candidates during the electoral period. As seen in Shepsle (1972), Page (1976), Glazer (1990), and others, candidates often have incentives to adopt ambiguous positions on matters that are relevant for voters' decision, which can be one of the reasons for the existence of an election with only "unknown" candidates. In this case, the factors that lead to freedom of choice constraining would be mainly linked to the strategies adopted by candidates - that is, the lack of their interest in revealing information and avoiding ambiguous discourses - and not with the potential absence of instruments to reveal it.

The following proposition explores which results can be expected when voters are highly tolerant with uncertainty about candidates positions on issues.

Proposition 4. If, for all $i \in I$, $\tau_i = \log N$, then $\nexists\ K \in Z$ with $K \neq I$. Consequently, $(M, I) \succsim_i^{\mathcal{Z}} (M', I)$ if $\#M \geq \#M'$.

Proof. Take the election I, where every voter is also a candidate. Then, for all $m \in I$, $\tau_i \geq H(\mathcal{P}_m^i)$, and, by Axiom TE, for all $m \in I$, it is also true that $m \in (I \cap K)$, which implies in $K \subseteq I$, and $I \subseteq K$. Since all individuals are "known", for any $M, M' \in Z$, $(M \cap I) = M$, and $(M' \cap I) = M'$, which, by the Known-Option-Based rule, results in $(M, I) \succsim_i^{\mathcal{Z}} (M', I) \Leftrightarrow \#M \geq \#M'$. □

What is being said through Proposition 4 is that, when voters' tolerance levels are maximal, i.e., when their tolerance level is equal to the entropy of an uniform distribution on X, then all individuals will be regarded as "known". Consequently, all candidates also will be "known", leading us to conclude that informational factors do not play any direct role to the comparisons of opportunity sets in terms of freedom of choice. In this case, the rule of Arlegi and Dimitrov (2005) reaches the same results of the axiomatic approach of Pattanaik and Xu (1990), where the only relevant information to the establishment of comparisons among sets is their cardinalities.

This theoretical formulation, however, also makes possible the following result, which seems to contradict a primal notion of liberty, i.e., that it depends on the existence of alternatives to be chosen, which is intimately related to Pattanaik and Xu's INS axiom.

Proposition 5. If $\succsim_i^{\mathcal{Z}}$ is the Known-Option-Based ordering, then, for any $M' \in Z$ such that $(M' \cap K') = \varnothing$, we have $(\{m\}, K) \succ_i^{\mathcal{Z}} (M', K')$, for any $m \in K$.

Proof. Take two elections with only one candidate, $\{m\}, \{m'\} \in Z$, such that $m \in K$, but $m' \notin K$. Then, $(\{m\} \cap K) = \{m\}$, and $(\{m'\} \cap K') = \varnothing$, which means that $\#(\{m\} \cap K) > \#(\{m'\} \cap K')$, and hence $(\{m\}, K) \succ_i^{\mathcal{Z}} (\{m'\}, K')$. Since $(\{m'\} \cap K') = \varnothing$, then the set $I \backslash (K' \cup \{m'\})$ denotes the set of all individuals that are "unknown" to individual i when he knows K', and that are not candidates in election $\{m'\}$. Define $B \subseteq I \backslash (K' \cup \{m'\})$ as any non-empty subset of individuals with this characteristics, and an election M' such that $M' = \{m\} \cup B$. The cardinality of $(M' \cap K')$ can be written as $\#(M' \cap K') = \#(\{m'\} \cap K') + \#(B \cap K')$, but since $B \subseteq I \backslash (K' \cup \{m'\})$, for any B, $(B \cap K') = \varnothing$ will hold, and hence $\#(M' \cap K') = \#(\{m'\} \cap K')$, which implies in $(\{m'\}, K') \sim_i^{\mathcal{Z}} (M', K')$. But if that is the case, by transitivity we have that $(\{m\}, K) \succ_i^{\mathcal{Z}} (M', K')$, which completes the proof. □

The above proposition states that any one-candidate election, observed that this candidate is "known", will provide more freedom of choice than any other possible election with only "unknown" competitors, no matter how many "unknown" alternatives those elections provide. This seems as a counterintuitive result at first glance, since a singleton election, regardless the informational factor, does not seem to provide any freedom of choice *at all* given that voters have only one alternative for choice. For instance, imagine a country that is being ruled by the same dictator for the last twenty years, but, in order to assure the legitimacy of his mandate, presidential elections are convoked. It is reasonable to believe that, for the majority of voters, this dictator will be considered as "known". Assume now that, afraid of a possible retaliations, or imagining that electoral results will be manipulated to corroborate the actual government, no other citizen decide to engage in the electoral contest, culminating in an election with only candidate. Given this situation, would people really experience more freedom of choice through this election than in a hypothetical one with many "unknown" candidates? In such a situation, it seems plausible to imagine that people would prefer the second election, loosing their strict preference for "known" candidates.

6 Concluding Remarks

During this essay were discussed some approaches of freedom of choice, focusing in two simple rules that have been proposed in the literature: the simple cardinality-based approach of Pattanaik and Xu (1999), and the known-option-based rule of Arlegi and Dimitrov (2005). Both rules are compared and their results analyzed within the context of political elections, where it is assumed that the electorate can be unsure about the ideological positions of candidates, i.e., the political platform that each competitor will adopt when in office, leading each voter to represent every available candidate as a lottery in this issue space.

To contribute to this discussion, it is suggested here that the entropy of the probability distribution used to describe each candidate in the issue space can be an useful theoretical apparatus to access if this competitor is seen as "known" or "unknown" to the voter. Also, it is assumed that voters can tolerate some amount of uncertainty and still consider the candidate as "known". Then, it is shown that the simple cardinality-based and the known-option-based rules will converge to the same results only if all candidates are "known", situation that can be feasible if the entropy of their probabilistic representations are small enough, or if voters have the highest degree of tolerance as possible, configuring a situation where information do not play any specific role to freedom of choice considerations. To the contrary, if voters are not completely tolerant with uncertainty regarding candidates' ideological positions, and if those candidates adopt some sort of maximum ambiguity strategy, where no relevant information is revealed by candidates to facilitate voters' choice, then all competitors will be classified as "unknown", culminating in an election with the smallest degree of freedom. Also, it is explored the counterintuitive possibility of an election with only one "known" candidate to provide more freedom than another election with multiple "unknown" competitors, leading to the discussion of how appropriate a total priority to knowledge can be to describe peoples' feelings toward liberty in political elections.

Clearly, both approaches are simple and do not depict the general case, but they introduce some interesting foundations to the freedom of choice and social choice discussion, and the role of uncertainty about alternatives to individuals' perception of freedom as well. Many further developments can be done, incorporating other features to the discussion, such as diversity, autonomous choice under uncertainty, and a deeper analysis of the idea of degrees of uncertainty, also trying to explore empirically the freedom concept and its consequences on politics and voters behavior.

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